Mathematical Vocabulary for Math 320

This document is not finished: as the class progresses new items will be added, and you should add items of your own. Check the textbook to see if you can find where these concepts are defined. Write the page numbers next to the definitions on this sheet.

1. Math 320

Linear independence

Vectors $\mathbf{v}_1, \cdots, \mathbf{v}_k$ are linearly independent if the only solution to $c_1\mathbf{v}_1 + \cdots + c_k\mathbf{v}_k = 0$ is $c_1 = \cdots = c_k = 0$.

To span

Vectors $\mathbf{v}_1, \cdots, \mathbf{v}_k$ *span a linear subspace L of* $\mathbb{R}^n$ if every vector in L is a linear combination of the vectors $\mathbf{v}_1, \cdots, \mathbf{v}_k$.

Linear subspace

If L is a set of vectors in $\mathbb{R}^n$, then L *is a linear subspace of* $\mathbb{R}^n$ if
(i) for any two vectors $\mathbf{u} \in L, \mathbf{v} \in L$ the sum $\mathbf{u} + \mathbf{v}$ also belongs to L, *and*
(ii) for any vector $\mathbf{v} \in L$ and any number c, the vector $c\mathbf{v}$ also belongs to L.

Solution space

The solution space of a set of homogeneous linear equations $A\mathbf{x} = \mathbf{0}$ is the set which consists of all vectors $\mathbf{x}$ which satisfy the equation. This notion is only used for homogeneous equations.

Basis

Vectors $\mathbf{v}_1$, ..., $\mathbf{v}_n$ form a *basis for a linear subspace L* if they are linearly independent, and if they span L.

Dimension

The *dimension of a linear subspace L* of $\mathbb{R}^n$ is the number of vectors in a basis for L. Note: in mathematical usage this word is always in the singular: you say "the dimension of L is 3," but not "L has three dimensions."

General Solution (for linear equations)

The general solution of a linear system of equations $A\mathbf{x} = \mathbf{b}$ is a formula containing a number of parameters such that any choice of the parameters gives you a solution to $A\mathbf{x} = \mathbf{b}$, and such that every solution can be found by choosing appropriate values of the parameters.

General Solution (for a differential equation)

The general solution of a linear differential equation

$$y^{(n)}(t) + p_1(t)y^{(n-1)}(t) + \cdots + p_{n-1}(t)y'(t) + p_n(t)y(t) = f(t)$$

is a formula containing a number of parameters (usually called c_1, c_2, etc.) such that any choice of the parameters gives you a solution to the equation, and such that every solution can be found by choosing appropriate values of the parameters.

2. Miscellaneous

The words ***formula*** and ***equation*** are frequently confused. Try getting them right to achieve that "I know what I'm talking about" air!

Formula — A group of symbols representing a mathematical object. E.g. "$2 + 3$," or "$\sqrt{x^2 + 1}$," or "$y''(t) + \sin(t)y(t)$".

Equation — A group of symbols expressing that two formulas are equal. E.g. "$(x - 1)(x + 1) = x^2 - 1$," or "$2x + y = 17$."
An equation always contains an equality sign. It is incorrect to call "$2x+3$" an equation; "$2x + 3$" is a formula.