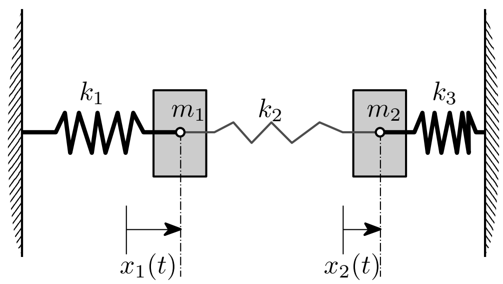
– MATH 320 –
TWO MASS PROBLEM



There's no gravity; two masses are connected to the walls by very strong springs, and they are connected to each other by a very weak spring. If we pull one mass to the right and let go, what will happen?

In math 320 the equations (coming from "$F = ma$" and Hooke's law) are in the book. In another class you would have to derive them (or see if the students can).

The answer, which is unintuitive, is that initially one mass will be oscillating. Since it is connected to the other mass by a weak spring it will make the other mass move. Slowly but surely all energy from the motion of the first mass will be transferred to the other. When that has happened the same thing happens, but with the roles of the two masses reversed, ad infinitum.

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1. Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

where a and b are two arbitrary numbers. Hint; compute $A\left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right]$ and $A\left[\begin{smallmatrix} 1 \\ -1 \end{smallmatrix}\right]$.

2. (See page 433 in the text book for a derivation of the equations governing the motion of the two masses in the drawing). Suppose that

$$m_1 = m_2 = 1,$$

and

$$k_1 = k_3 = 10202, \text{ and } k_2 = 101.$$

So the two masses are equal, and the two outer springs are about a hundred times as strong as the one connecting the two masses.

If $x_1(t)$ and $x_2(t)$ are the deviation from equilibrium for both masses, then find

$$\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

assuming the initial conditions

$$\boldsymbol{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \text{ and } \boldsymbol{x}'(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

3. Use a graphing calculator, or better, a graphing program, to plot your solutions $x_1(t)$ and $x_2(t)$ for $0 < t < 2\pi$.