Some Wronskians

§5.2 – Problem 13.

$$\begin{aligned} W &= \begin{vmatrix} e^x & e^{-x} & e^{-2x} \\ e^x & -e^{-x} & -2e^{-2x} \\ e^x & e^{-x} & 4e^{-2x} \end{vmatrix} \\ &= e^{x-x-2x} \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -2 \\ 1 & 1 & 4 \end{vmatrix} \\ &= e^{-2x} \begin{vmatrix} 1 & & \\ 0 & -2 & \\ 0 & 0 & 3 \end{vmatrix} \\ &= -6e^{-2x} \end{aligned}$$

§5.2 – Problem 15.

$$\begin{aligned} W &= \begin{vmatrix} e^x & xe^x & x^2e^x \\ e^x & (x+1)e^x & (x^2+2x)e^x \\ e^x & (x+2)e^x & (x^2+4x+2)e^x \end{vmatrix} \\ &= e^{3x} \begin{vmatrix} 1 & x & x^2 \\ 1 & x+1 & x^2+2x \\ 1 & x+2 & x^2+4x+2 \end{vmatrix} \\ &= e^{3x} \begin{vmatrix} 1 & & \\ 0 & 1 & 2x \\ 0 & 2 & 4x+2 \end{vmatrix} \\ &= e^{3x} \begin{vmatrix} 1 & & \\ 0 & 1 & \\ 0 & 0 & 2 \end{vmatrix} \\ &= 2e^{3x}. \end{aligned}$$

§5.2 – Problem 19.

$$W = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$$

(factor out an x from first row and last column)

$$\begin{aligned} &= x \cdot x \begin{vmatrix} 1 & x & x \\ 1 & 2x & 3x \\ 0 & 2 & 6 \end{vmatrix} \\ &= x^2 \begin{vmatrix} 1 & & \\ 0 & x & 2x \\ 0 & 2 & 6 \end{vmatrix} \\ &= 6x^3 \end{aligned}$$