MATH 319
Second Midterm Exam–Spring 1998

Your Name:					
1	**2**	**3**	**4**	**5**	**Total**

1. Decide if $x = 0$ is an ordinary point, regular singular point or irregular singular point for each of the following two differential equations:

(a) $$\left(x^2 + \frac{1}{x}\right)y''(x) + xy'(x) + \left(3 - 2x + x^2\right)y(x) = 0.$$

(b) $$x^3 y''(x) + xy'(x) + \left(3 - 2x + x^2\right)y(x) = 0.$$

2. Solve the following differential equation by the power series method

$$\begin{cases} y''(x) - \dfrac{2}{1+x^2}\,y(x) = 0 \\ y(0) = 1; \quad y'(0) = 0. \end{cases}$$

Simplify the recurrence relation as much as you can. Give a formula for the coefficient of x^n in the solution $y(x)$.

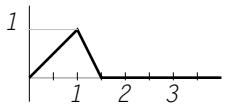
3. (a) Find ***one*** nonzero solution of the following differential equation by the Fröbenius method:

$$y''(x) + \left(\frac{4}{x} - x\right)y'(x) + \lambda y(x) = 0\,.$$

Your solution should contain: the indicial equation, the recurrence relation, a formula for the n^{th} coefficient in your series solution.

(b) For which values of λ does the series solution terminate (i.e. when does the series have only finitely many terms?)

4. (a) Compute the Laplace transform of $f(x)$ given by the graph



(b) Find the inverse Laplace transform of $F(s) = \dfrac{s}{\left(s^2 + a^2\right)^2}$ (*hint*: consider $\frac{d}{ds}\left(s^2 + a^2\right)^{-1}$).

5. Solve the following initial value problem using the Laplace transform.

$$\begin{cases} y^{\text{iv}} - 5y'' + 4y = \sin t \\ y(0) = y'(0) = y''(0) = 0 \\ y'''(0) = 1 \end{cases}$$

(Partial answers: the Laplace transform of the solution; Its partial fraction expansion; the coefficients in the expansion; the inverse tranform.)