1(a) Linear 1st order inhomogeneous. Multiplication with the integrating factor $e^{\arctan x}$ leads to $ye^{\arctan x} = \int \arctan x\, e^{\arctan x} \frac{dx}{1+x^2}$. To do the integral let $u = \arctan x$ and you get $\int ue^u du = ue^u - e^u + C$ (integrate by parts), so that

$$y = \arctan x - 1 + Ce^{-\arctan x}$$

Put $x = 0$ to find that $C = K + 1$

1(b) The equation is separable. Separate variables to get $\int \frac{dy}{y^3} = \int \frac{dx}{1+x^2}$, and hence $\frac{-1}{2y^2} = \arctan x + C$. The initial condition $y(0) = K$ implies $C = \frac{-1}{2K^2}$ and thus

$$y = \frac{K}{\sqrt{1 - 2K^2 \arctan x}}$$

1(c) The solution is the sum of a solution to the homogeneous equation and a particular solution. The characteristic equation is $r^2 - 3r - 4 = 0$ which has roots $r = -1,\ r = 4$ and the solution to the homogeneous equation is $y_h = Ae^{4x} + Be^{-x}$
To find a particular solution one tries $y_p = Ce^{-x}\sin 2x + De^{-x}\cos 2x$. You then get

$$y_p''(x) - 3y_p'(x) - 4y_p(x) = (-4C + 10D)e^{-x}\sin 2x + (-10C - 4D)e^{-x}\cos 2x$$

For this to be equal to $e^{-x}\sin 2x$ we have to impose the following conditions for C and D: $-4C + 10D = 1$ and $-10C - 4D = 0$. The solution is $C = -\frac{1}{29},\ D = \frac{5}{58}$.
So the general solution is:

$$y_p = Ae^{4x} + Be^{-x} - \tfrac{1}{29}e^{-x}\sin 2x + \tfrac{5}{58}e^{-x}\cos 2x$$

To satisfy the initial conditions you have to solve the following equations in A and B: $A + B = \frac{5}{58} + K,\ 4A - B = \frac{9}{58}$ with solution $A = \frac{14}{290} + \frac{1}{5}K,\ B = \frac{11}{290} + \frac{4}{5}K$

2. Use reduction of order. Put $y(x) = \left(1+x^2\right)u(x)$, and get

$$\left(1+x^2\right)u''(x) + 4xu'(x) = 0.$$

Put $v(x) = u'(x)$ and solve the resulting equation for $v(x)$ with result

$$\ln v(x) = -\int \frac{4x\,dx}{1+x^2} = -2\ln\left(1+x^2\right) = \ln\left(1+x^2\right)^{-2},$$

so that

$$u'(x) = v(x) = \frac{1}{\left(1+x^2\right)^2}$$

Hence the general solution is

$$y(x) = A\left(1+x^2\right)u(x) + B\left(1+x^2\right) = A\left(1+x^2\right)\int \frac{dx}{\left(1+x^2\right)^2} + B\left(1+x^2\right)$$