MATH 319 – DIFFERENTIAL EQUATIONS FIRST MIDTERM EXAM. FRIDAY OCTOBER 11, 1996, 8:50AM-10:00AM.

1. Find the general solution of the following differential equations.

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{y}{1+x} = 1+x,\tag{1}$$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = xe^{4x} + 18x^2.$$
 (2)

2. Which of the following two equations is exact?

$$(1+6xy)\frac{\mathrm{d}y}{\mathrm{d}x} + 5 + 2x + 3y^2 = 0, \tag{3}$$

$$6xy\frac{\mathrm{d}y}{\mathrm{d}x} + x^2y - 17 = 0.$$
 (4)

Of these two equations, solve the exact equation (write the solution as y = y(x)!)

3. The functions $y_1(x) = 1 + x^2$ and $y_2(x) = x + x^3$ are solutions of the linear second order homogeneous ordinary differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - \frac{4x}{1+x^2} \frac{\mathrm{d}y}{\mathrm{d}x} + \frac{6x^2 - 2}{(1+x^2)^2}y = 0.$$

Do y_1 and y_2 form a fundamental set of solutions? Motivate your answer.