234 Review Sheet 2

1. Classify the following quadratic forms:

   (a) $x^2 - 4xy + 5y^2$
   (b) $3x^2 + xy - 2y^2$
   (c) $x^2 + 10xy + y^2$
   (d) $4x^2 - xy$
   (e) $6xy$
   (f) $(x - y)(x - 2y)$

2. Consider the following functions $f(x, y)$, do each of the following *in order*

   - Find the level set $f(x, y) = 0$
   - Using the implicit function theorem find all points on the level set such that, around those points, the graph of the level set is the graph of a function $y = g(x)$.
   - At every such point (x_0, y_0), find $g'(x)$ (NOTE $g(x)$ may be different at different points. There may not be an "explicit formula" for $g(x)$, use the implicit function theorem for this question)

   (a) $f(x, y) = x + y$
   (b) $f(x, y) = x^2 + y^2$
   (c) $f(x, y) = x^2 - y^2$

3. Consider the following functions $f(x, y, z)$, do each of the following *in order*

   - Find the level set $f(x, y, z) = 5$
   - Using the implicit function theorem find all points on the level set such that, around those points, the graph of the level set is the graph of a function $y = g(x, z)$.
   - At every such point (x_0, y_0, z_0), find $\frac{\partial g}{\partial x}$ and $\frac{\partial g}{\partial z}$ (NOTE $g(x, z)$ may be different at different points. There may not be an "explicit formula" for $g(x)$, use the implicit function theorem for this question)
   - Find an equation for the tangent plane to the level set $f(x, y, z) = 5$ at the point p.

   (a) $f(x, y, z) = x^2 + 2y^2 + 3z^2$, $p = (0, 1, 1)$
   (b) $f(x, y, z) = \ln(xyz) + y$, $p = (1/5, 5, 1)$
   (c) $f(x, y, z) = e^x + e^y + e^z$, $p = (0, \ln 2, \ln 2)$

4. For each of the specified functions $f(x, y)$ and points p do the following:

   - Convert the function to a function $g(r, \theta)$ in polar coordinates
   - Find the partal derivatives of f with repect to x, y, r, θ.
   - Find the tangent plane to the surface $z = f(x, y)$ at the point p.
   - Find and draw the level sets of $z = f(x, y)$ at $z = 0, 2, -1$.
   - Suppose $x(t) = t + \ln(t)$ and $y(t) = \cos(t)$, and find $\frac{df}{dt}$.

(a) $f(x,y) = x^2 + y^2$, $p = (3,2,13)$

(b) $f(x,y) = xy$, $p = (1,1,1)$

(c) $f(x,y) = \sin(x)$, $p = (0,0,0)$

(d) $f(x,y) = e^x y$, $p = (1,1,e)$

5. Suppose you are in a spaceship nearing a black hole. The gravitational force of the black hole is given by

$$G(x,y,z) = \frac{1}{x^2 + y^2 + z^2 - 1}$$

For $x^2 + y^2 + z^2 > 1$. All points such that $x^2 + y^2 + z^2 \leq 1$ are inside the "event horizon" of the black hole and can never escape. To avoid getting turned into spaghetti, you want to move away from the black hole as quickly as possible in a way that the gravitational force decreases the most. If your spaceship is sitting at the point (3,2,3), what dircetion should you pilot your spaceship in?