math 234 final exam, fall 2013.

(1) Let $f(x, y) = xe^{x-2y}$.
  (a) Find the linear approximation of $f(x, y)$ near $x = 2, y = 1$.
  (b) Use the linear approximation to approximate $f(1.99, 1.02)$.

(2) Suppose we are given a function $f(x, y)$. Define $g(u, v) = f(u \ln v, u + v)$. In the following questions your answer may contain the variables u and v as well the partial derivatives of f with respect to x and/or y.
  (a) Compute $\frac{\partial g}{\partial u}$.
  (b) Compute $\frac{\partial^2 g}{\partial u \partial v}$.

(3) (a) Find all critical points of the function $f(x, y) = x^2 - 3y^2 + y^3$;
  (b) Use the second derivative test to tell which of the critical points are local maxima, minima, or saddle points. For any saddle points find the tangent lines to the level set.

(4) Use the method of Lagrange multipliers to find the largest and smallest values that xy^2 can have if $x^2 + y^2 = 3$.

(5) Let V be the volume under the graph of

$$z = \frac{1}{(x + y)^2}$$

above the rectangle in which $3 \le x \le 6$ and $0 \le y \le 2$.
  (a) Which integral do you have to compute to find V?
  (b) Compute V.

(6) Consider the integral

$$I = \iint_{\mathcal{R}} f(x, y)\, dA = \int_1^5 \int_{(x-1)/2}^{\sqrt{x-1}} f(x, y)\, dy\, dx$$

  (a) Draw the domain $\mathcal{R}$
  (b) Find the integration bounds that appear when you rewrite the integral in the form

$$I = \int_{\cdots}^{\cdots} \int_{\cdots}^{\cdots} f(x, y)\, dx\, dy$$

(7) Let $\mathcal{R}$ be the three dimensional region given by $x \ge 0$, $y \ge x$, and $x^2 + y^2 \le 4$, and $0 \le z \le x^2 + y^2$.
  (a) Describe $\mathcal{R}$ in cylindrical coordinates (r, θ, z).
  (b) Compute the average of z in $\mathcal{R}$.

(8) Let $\mathcal{R}$ be the three dimensional region inside the sphere $x^2 + y^2 + z^2 \leq 9$ that is given by $x \geq 0$, $y \geq 0$, $z \geq 0$, and $y \geq x$.

(a) Make a drawing that describes the spherical coordinates (ρ, ϕ, θ) of a point and describe the region $\mathcal{R}$ in spherical coordinates.

(b) Write the integral

$$I = \iiint_{\mathcal{R}} x \, dV$$

in terms of spherical coordinates. Do not compute the integral.

(9) Let A, B, and C be the three points $A(0,0)$, $B(1,1)$, $C(2,0)$.

Let $\vec{\boldsymbol{F}}$ be the vectorfield $\vec{\boldsymbol{F}}(x,y) = \binom{-1}{1}$.

(a) Compute $\int_{\mathcal{C}} \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{T}} \, ds$ where $\mathcal{C}$ is the curve that consists of the line segment AB followed by the line segment BC (i.e. $\mathcal{C}$ is the top of the triangle ABC, oriented from left to right.)

(b) Compute $\int_{\mathcal{C}} \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{N}} \, ds$, where $\vec{\boldsymbol{N}}$ is the unit normal to $\mathcal{C}$ that points upward.

(c) Use Green's theorem to compute the line integral

$$I = \int_{\mathcal{T}} \vec{\boldsymbol{G}} \cdot \vec{\boldsymbol{T}} \, ds$$

where $\mathcal{T}$ is the triangle ABC with counter-clockwise orientation, and $\vec{\boldsymbol{G}}$ is the vector field

$$\vec{\boldsymbol{G}}(x,y) = \begin{pmatrix} 0 \\ x \end{pmatrix}.$$