Math 234, Fall 2013 Worksheet #2

## The cycloid

Read the description of the cycloid in the text (Ch2, §5). Suppose a point X(t) moves on a cycloid, where  $\theta = \omega t$ , so that

$$\vec{x}(t) = \begin{pmatrix} R\theta - R\sin\theta\\ R - R\cos\theta \end{pmatrix} = \begin{pmatrix} R\omega t - R\sin\omega t\\ R - R\cos\omega t \end{pmatrix}$$

Here R is the radius of the wheel, and  $\omega$  is the rate at which the wheel turns. We will assume for simplicity in this problem that R = 1 and that  $\omega = 1$ .

**1**. Compute the velocity and acceleration vectors.

2. *True or False*: the acceleration always points toward the center of the wheel?

**3**. Find the slope of the tangent to the curve at the point X(t).

**4**. *True or False:* the velocity is always tangential to the wheel (i.e. perpendicular to the segment connecting the point and the center of the wheel)?

5. Find an integral for the length of one arc of the cycloid (i.e. the piece with  $0 \le t \le 2\pi$ .)