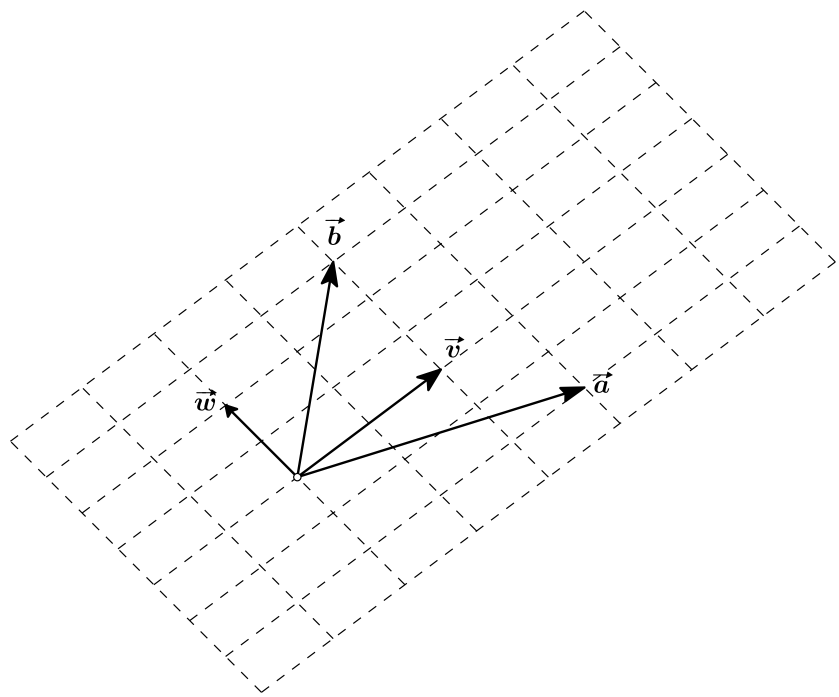
1. Linear combinations of vectors

If $\vec{a}$ and $\vec{b}$ are two vectors, then the expression $s\vec{a} + t\vec{b}$, where s and t are numbers, is called a ***linear combination*** of $\vec{a}$ and $\vec{b}$.



Problems

1. Draw the vectors $2\vec{v} + \frac{1}{2}\vec{w}$, $\quad -\frac{1}{2}\vec{v} + \vec{w}$, $\quad$ and $\frac{3}{2}\vec{v} - \frac{1}{2}\vec{w}$
2. Find real numbers s, t such that $s\vec{v} + t\vec{w} = \vec{a}$.
3. Find real numbers p, q such that $p\vec{v} + q\vec{w} = \vec{b}$.

2. Vectors and Coordinates

Two observers, A (Albert) and B (Bernice), are describing points in the plane. They both choose a special point, which they call the origin O.

Observer A chooses two vectors $\vec{e}_1$ and $\vec{e}_2$, and makes sure that $\vec{e}_1$ and $\vec{e}_2$ form an ***orthonormal set of vectors.*** This means by definition that

- both vectors $\vec{e}_1$ and $\vec{e}_2$ have length 1, and
- the vectors $\vec{e}_1$ and $\vec{e}_2$ are perpendicular ($\vec{e}_1 \perp \vec{e}_2$)

A pair of vectors $\{\vec{e}_1, \vec{e}_2\}$ with these properties is sometimes also called an ***orthonormal basis.***

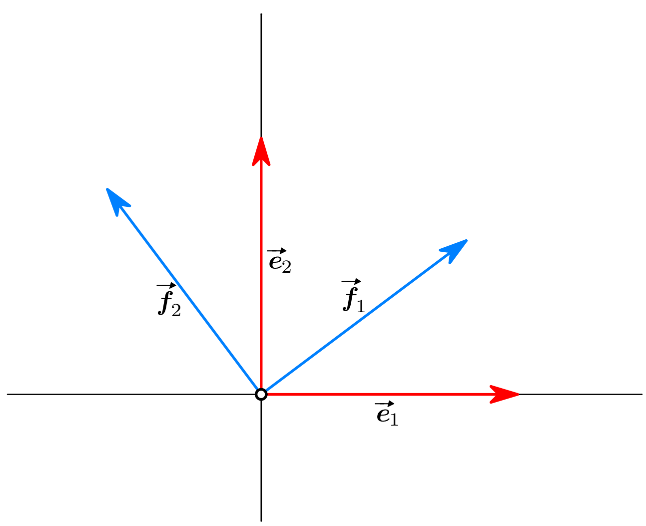
Observer A uses these two vectors to assign (x_1, x_2) coordinates to points in the plane. A does this by saying that (x_1, x_2) are the coordinates of the point P if

$$\overrightarrow{OP} = x_1\vec{e}_1 + x_2\vec{e}_2.$$

Observer B, chooses a different orthonormal set of vectors $\vec{f}_1$ and $\vec{f}_2$, and assigns coordinates (y_1, y_2) to the same point P if

$$\overrightarrow{OP} = y_1\vec{f}_1 + y_2\vec{f}_2.$$

Note that while A and B have different coordinate axes, they did choose the same point O to be the origin. ***How can you translate between A and B's coordinates?***



Problems

Suppose that B's first basis vector $\vec{f}_1$ can be written in terms of A's basis vectors $\vec{e}_1$, $\vec{e}_2$ as

$$\vec{f}_1 = \frac{4}{5}\vec{e}_1 + \frac{3}{5}\vec{e}_2.$$

1. Draw the two coordinate axes that B uses.

2. According to observer A, P is the point with coordinates $(1, 1)$. Draw the point, and, using your drawing, estimate the coordinates the observer B would assign to P.

3. Write $\vec{f}_1$ and $\vec{f}_2$ in terms of $\vec{e}_1$ and $\vec{e}_2$.

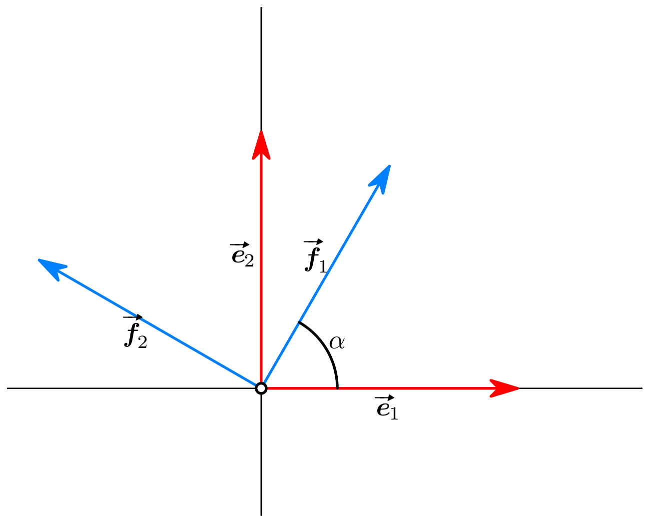
4. Write $\vec{e}_1$ and $\vec{e}_2$ in terms of $\vec{f}_1$ and $\vec{f}_2$.

5. Express x_1 and x_2 in terms of y_1 and y_2.

6. Express y_1 and y_2 in terms of x_1 and x_2. Compute observer B's coordinates for the point P, and compare the answer with your estimate from question 1.

7. If you know that $x_1^2 + x_2^2 = 5$, then how much is $y_1^2 + y_2^2$?

> Suppose now that, instead of $\vec{f}_1 = \frac{3}{5}\vec{e}_1 + \frac{4}{5}\vec{e}_2$,
>
> we are given that the angle between $\vec{e}_1$ and $\vec{f}_1$ is α.



8. Answer questions 1–7 in this, more general, setting.