Problem 8.

If $F(r,\theta) = f(r\cos\theta, r\sin\theta)$ then what are F_θ and $F_{\theta\theta}$.

We apply the chain rule:

$$\frac{\partial F}{\partial \theta} = \frac{\partial f(\overbrace{r\cos\theta}^{=x}, \overbrace{r\sin\theta}^{=y})}{\partial \theta}$$

$$= \frac{\partial f}{\partial x} \cdot \frac{\partial r\cos\theta}{\partial \theta} + \frac{\partial f}{\partial y} \cdot \frac{\partial r\sin\theta}{\partial \theta}$$

$$= -r\sin\theta \frac{\partial f}{\partial x} + r\cos\theta \frac{\partial f}{\partial y}.$$

Note that here $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x}(r\cos\theta, r\sin\theta)$.

To get $\frac{\partial^2 F}{\partial \theta^2}$ differentiate again:

$$\frac{\partial^2 F}{\partial \theta^2} = \frac{\partial}{\partial \theta}\left(-r\sin\theta \frac{\partial f}{\partial x} + r\cos\theta \frac{\partial f}{\partial y}\right)$$

$$= \frac{\partial}{\partial \theta}\left(-r\sin\overset{1}{\theta} \frac{\partial f}{\partial x}(r\cos\overset{2}{\theta}, r\sin\overset{3}{\theta}) + r\cos\overset{4}{\theta} \frac{\partial f}{\partial y}(r\cos\overset{5}{\theta}, r\sin\overset{6}{\theta})\right)$$

There are six θ's so we will get <u>six terms</u> when we differentiate with respect to θ.

The result is

$$\frac{\partial^2 F}{\partial \theta^2} = \overbrace{-r\cos\theta \frac{\partial f}{\partial x}}^{1} \overbrace{- r\sin\theta \cdot \frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial r\cos\theta}{\partial \theta}}^{2}$$

$$\overbrace{-r\sin\theta \frac{\partial^2 f}{\partial x \partial y} \cdot \frac{\partial r\sin\theta}{\partial \theta}}^{3} \overbrace{- r\sin\theta \frac{\partial f}{\partial y}}^{4}$$

$$\overbrace{+r\cos\theta \cdot \frac{\partial^2 f}{\partial y \partial x} \cdot \frac{\partial r\cos\theta}{\partial \theta}}^{5} \overbrace{+ r\cos\theta \cdot \frac{\partial^2 f}{\partial y^2} \cdot \frac{\partial r\sin\theta}{\partial \theta}}^{6}$$

$$= -r\cos\theta \frac{\partial f}{\partial x} - r\sin\theta \frac{\partial f}{\partial y}$$

$$+ r^2 \sin^2\theta \frac{\partial^2 f}{\partial x^2} - 2 \cdot r^2 \sin\theta \cos\theta \frac{\partial^2 f}{\partial x \partial y}$$

$$+ r^2 \cos^2\theta \frac{\partial^2 f}{\partial y^2}$$