5 Two critical points for xye^{-ax-by}:

$(0,0)$ and $\left(\frac{1}{a}, \frac{1}{b}\right)$

6 One critical point for $x^a y^b e^{-x-y}$

namely at $(x,y) = (a,b)$.

(There could be critical points at $(0,0)$ and on the x- or y-axis depending on the values of a, b)

7a $\frac{\partial f}{\partial x} = Axy + B\cos x$

$\frac{\partial f}{\partial y} = x^2 + \sin y$.

If f exists then Clairaut says $f_{xy} = f_{yx}$,

so $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial\, Axy + B\cos x}{\partial y} = Ax$

$\frac{\partial^2 f}{\partial x \partial y} = 2x$.

Conclusion f does not exist, unless $A=2$.

B can be any thing.

Compute f when $A=2$.

$$\frac{\partial f}{\partial x} = 2xy + B\cos x \implies$$

$$f = \int (2xy + B\cos x)\,dx$$

$$= x^2 y + B\sin(x) + \underline{C(y)}$$

Constant, meaning, "does not depend on x". Could depend on y!

This function has

$$\frac{\partial f}{\partial y} = x^2 + C'(y) \overset{?}{=} x^2 + \sin y.$$

So $C'(y) = \sin y \implies C(y) = -\cos y + C$

truly constant depends neither on x nor on y

We find

$$\underline{f(x,y) = x^2 y + B\sin x - \cos y + C}$$