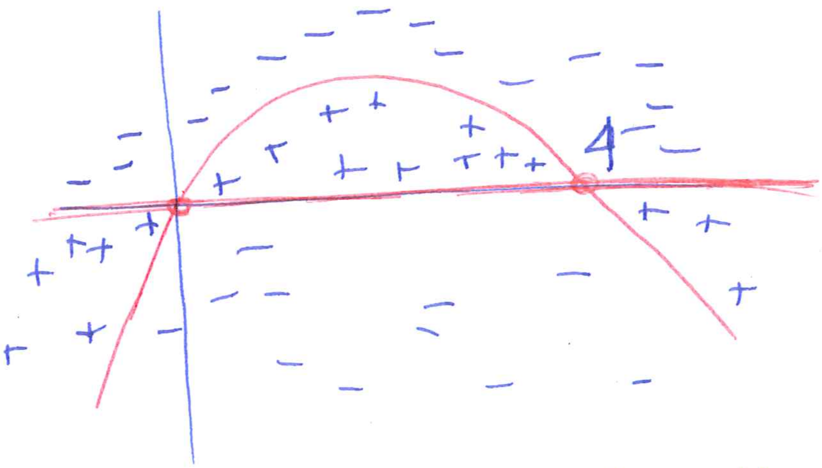
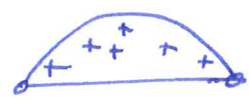
1b $f(x,y) = (4x - x^2 - y)\,y$

$= 4xy - x^2y - y^2$

Zero set: $y = 0$ or $y = 4x - x^2$



Saddle points at $(0,0)$ and $(4,0)$.

One local maximum in the region 

$f_x = 4y - 2xy = 2y(2-x)$

$f_y = 4x - x^2 - 2y$

Critical points: $y=0$ and $4x - x^2 = 0$ i.e. $(0,0)$, $(4,0)$

$x = 2$ and $8 - 2^2 - 2y = 0$
which leads to $(2,2)$

$$\boxed{\begin{array}{ccccc} (0,0) & , & (2,2) & , & (4,0) \\ \text{Saddle} & & \text{local max} & & \text{Saddle} \end{array}}$$

1b Taylor & 2nd derivative test

$$f_{xx} = -2y$$

$$f_{xy} = 4-2x$$

$$f_{yy} = -2$$

At (0,0):

$$f(\Delta x, \Delta y) = \frac{1}{2}\left\{ 0\cdot(\Delta x)^2 + 2\cdot 4\,\Delta x\,\Delta y - 2(\Delta y)^2 \right\} + \cdots$$

$$= 4\,\Delta x\,\Delta y - (\Delta y)^2$$

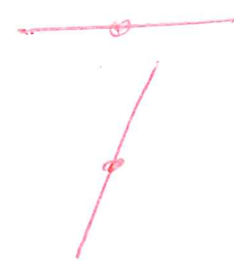
$$= (4\,\Delta x - \Delta y)\cdot \Delta y$$

Indefinite $\Rightarrow$ a saddle

The two tangents are

$\Delta y = 0$

$\Delta y = 4\,\Delta x$

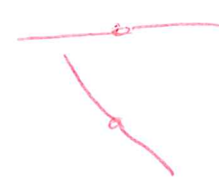


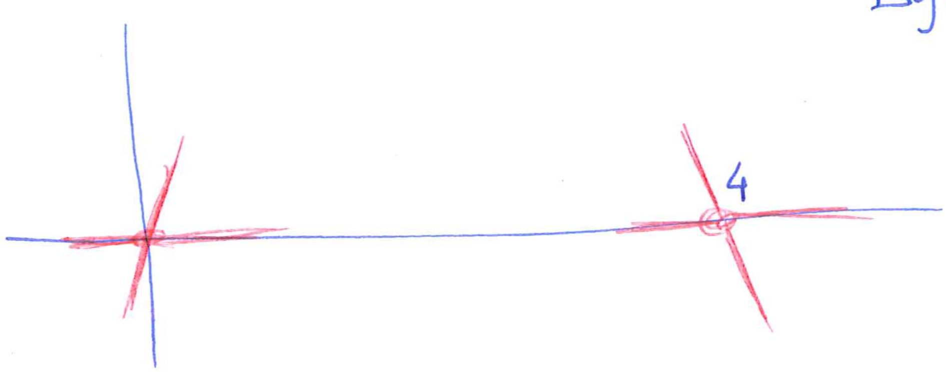
At (4,0)

$$f(4+\Delta x, \Delta y) = -4\,\Delta x\,\Delta y - (\Delta y)^2 = -\Delta y(\Delta y + 4\Delta x)$$

Again saddle with tangents $\Delta y = 0$

$\Delta y = -4\,\Delta x$





At $(2,2)$:

$$f(2+\Delta x, 2+\Delta y) =$$

$$= f(2,2) + \frac{1}{2}\left\{-2\cdot 2\,(\Delta x)^2 + 2\cdot(4-2\cdot 2)\,\Delta x\,\Delta y - 2(\Delta y)^2\right\} + \cdots$$

$$= f(2,2) \underbrace{-\,2\,(\Delta x)^2 - 2(\Delta y)^2} + \cdots$$

The 2^{nd} order terms are negative definite, so f has a local maximum at $(2,2)$.