1a Taylor expansion and Second Derivative Test.

Since $f(x,y) = 4xy - x^2y - y^3$ we have

$$f_x = 4y - 2xy \qquad f_y = 4x - x^2 - 3y^2$$

$$f_{xx} = -2y \qquad f_{xy} = 4 - 2x \qquad f_{yy} = -6y.$$

The Taylor expansions of second order at the critical points therefore are:

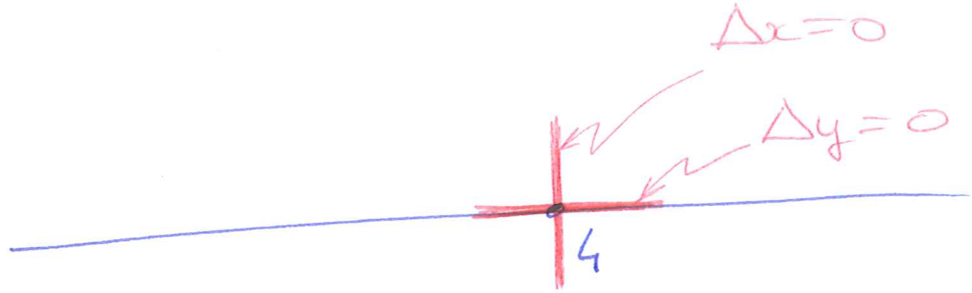
$$f(4+\Delta x,\ 0+\Delta x) = \overbrace{f(4,0)}^{=0} + \overbrace{f_x(4,0)\,\Delta x + f_y(4,0)\,\Delta y}^{=0 \text{ because } (4,0) \text{ is critical point}}$$

$$+ \frac{1}{2}\Big(\overbrace{f_{xx}(4,0)}^{=-0}(\Delta x)^2 + 2\underbrace{f_{xy}(4,0)}_{-8}\,\Delta x\,\Delta y + \underbrace{f_{yy}(4,0)}_{=0}(\Delta y)^2\Big)$$

$$+ \cdots$$

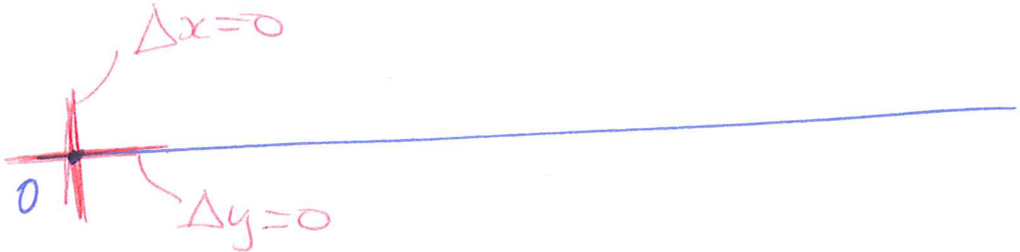
$$= -8\,\Delta x\,\Delta y$$

Thus $(4,0)$ is a saddle point and the tangents to the two curves that make up the zero set near $(4,0)$ are $\Delta x = 0$ and $\Delta y = 0$



At $(0,0)$ we get

$f(\Delta x, \Delta y) = +8\,\Delta x\,\Delta y$ also a saddlepoint.



At $(2, \tfrac{1}{2}\sqrt{3})$ we have

$$f(2+\Delta x, \tfrac{1}{2}\sqrt{3}+\Delta y) = f(2,\tfrac{1}{2}\sqrt{3}) + \frac{1}{2}\Big[\overset{f_{xx}(\dots)}{-\sqrt{3}}\,(\Delta x)^2 + \overset{f_{xy}(2,\frac{1}{2}\sqrt{3})=0}{0}\,\Delta x\,\Delta y - \underset{f_{yy}}{\sqrt{3}}\,(\Delta y)^2\Big]$$

$$= f(2,\tfrac{1}{2}\sqrt{3}) \underbrace{- \tfrac{1}{2}\sqrt{3}\,(\Delta x)^2 - \tfrac{1}{2}\sqrt{3}\,(\Delta y)^2} + \dots$$

Therefore $(2,\tfrac{1}{2}\sqrt{3})$ is a local maximum.