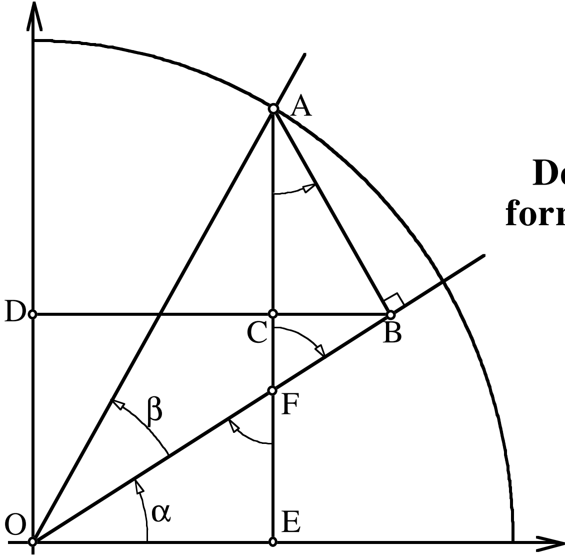

Math 171--Fall 1997
Angenent

Derivation of the addition formulas for Sine and Cosine

How to make this drawing:
First draw the circle, then angles α and β. Then draw AB so that OBA is a right triangle. Next, draw BD parallel to the *x*-axis and AE parallel to the *y*-axis.

Assume the circle has radius 1, and that its center is the origin.

Then $\sin(\alpha+\beta) = AE = AC+CE$.

Computation of AC:

OAB is a right triangle, with OA=1.
Hence $AB = 1\times \sin\beta = \sin\beta$.
The angles α and ∠OFE are complementary, so ∠OFE=90-α.
The angles ∠OFE and ∠AFB are equal, so ∠AFB=90-α.
The angles ∠AFB and ∠BAF are also complementary, so ∠BAF=90-∠AFB=90-(90-α) = α.
Now we know one angle and one side of the right triangle ABC, namely, ∠BAF= α, and AB = sin β.
Therefore we get <u>AC=cos α sin β</u>.

Computation of CE.

EC equals OD.
OBD is a right triangle.
The hypothenuse of OBD is OB, which is also the adjacent side of the right triangle OBA. Hence OB = cos β.
The angle ∠OBD equals the angle α since BD is parallel to the x-axis.
Now we know one angle and one side of the right triangle OBD, namely, ∠OBD= α, and OB = cos β.
Therefore we get

EC =OD = OB sin (∠OBD)
<u>=cos β sin α.</u>

Add the results of these two computations together, and you get

$$\mathbf{\sin(\alpha+\beta) = \sin\alpha\ \cos\beta + \sin\beta\ \cos\alpha}$$

As an exercise, try getting the formula for cos (α+β) in the same way.
Hint: cos (α+β) = OE = CD= BD-BC; now use the same two right triangles ABC and OBD.