**Math 570** **Fall 2016**

## Take-home Final

Instructions: For this exam, you are welcome to consult your textbook or class notes. You should not use other resources. You should work on these problems on your own and submit your own solutions. These solutions can either be submitted to me directly or by e-mail. The exam is due on Dec. 22nd at 4:45PM.

1. Show that there is no set $\mathcal{A}$ with the property that for every set there is some member of $\mathcal{A}$ that dominates it.

2. Assume that $S$ is a function with domain $\omega$ such that $S(n) \subseteq S(n^+)$ for each $n \in \omega$. (Thus $S$ gives an increasing sequence of sets.) Assume that $B$ is a subset of $\bigcup_{n\in\omega} S(n)$ such that for every infinite subset $B'$ of $B$ there is some $n$ for which $B' \cap S(n)$ is infinite. Show that $B$ is a subset of some $S(n)$.

3. Assume that $A$ is an infinite set. Show that $A \approx \mathrm{Sq}(A)$ (check the textbook for the definition of $\mathrm{Sq}(A)$).

4. Assume that $\lambda$ is an infinite cardinal number and that $2 \leq \kappa \leq \lambda$. Show that $\kappa^\lambda = 2^\lambda$.

5. Assume that two well-ordered structures are isomorphic. Show that there can be only 1 isomorphism from the first onto the second.

6. Show that if $R$ and $R^{-1}$ are both well-orderings of the same set $S$, then $S$ is finite.

7. (Transfinite induction schema) Let $\phi(x)$ be a formula and show that the following holds:

   Assume that for any ordinal $\alpha$, we have $((\forall x \in \alpha)\phi(x)) \Rightarrow \phi(\alpha)$. Then $\phi(\alpha)$ holds for any ordinal $\alpha$.