Math 275 **Review Sheet for Exam #3**

The third exam will be given on Friday, December 1, during the regular class period, 11:00 – 11:50 AM, in B139 Van Vleck Hall.

The exam will focus on material from chapter 5 in the text. This handout is intended to help you to study for the exam. However, you may be tested on any of the material we have covered. Just because an item does not appear on this sheet, it does not mean that you will not be asked about it. In particular, you should know how to do the problems that have been assigned for homework.

Summary of topics: You should know the precise statements of the definitions and theorems, and also how to apply the techniques of integration.

(1) **First fundamental theorem of calculus**

Let f be a function that is Riemann integrable on the interval $[a, b]$. Let $c \in [a, b]$. For $x \in [a, b]$, put $F(x) = \int_c^x f(t)\,dt$. Then

(a) The function F is continuous at every point of $[a, b]$.

(b) At every point $x \in [a, b]$ at which f is continuous, the function $F(x)$ is differentiable, and for such x we have $F'(x) = f(x)$.

(2) **The primitive of a function**

If f is a function defined on an interval $[a, b]$, a *primitive* (or *antiderivative* or *indefinite integral*) for f is any function F such that $F'(x) = f(x)$ for every $x \in [a, b]$.

(3) **Second fundamental theorem of calculus**

Assume that f is continuous on the interval $[a, b]$, and that P is any primitive of f. Then $\int_a^b f(t)\,dt = P(b) - P(a)$.

(4) **Zero-derivative theorem**

If f is continuous on a closed interval $[a, b]$ and differentiable at every point of the open interval (a, b), and if $f'(x) = 0$ for every $x \in (a, b)$, then f is constant on the interval $[a, b]$.

(5) **Integration by substitution**

$$\int_a^b G\big(f(t)\big)\, f'(t)\,dt = \int_{f(a)}^{f(b)} G(u)\,du.$$

(6) **Integration by parts**

$$\int_a^b f(x)\, g'(x)\, dx = f(b)\, g(b) - f(a)\, g(a) - \int_a^b f'(x)\, g(x)\, dx.$$

Results you should know how to prove:

(1) How the second fundamental theorem of calculus follows from the first fundamental theorem and the zero derivative theorem.

(2) How to prove the formula for integration by substitution.

(3) How to prove the formula for integration by parts.

(4) How to prove that the function $F(x) = \displaystyle\int_0^x \frac{\sin(t)}{\sqrt{t}}\, dt$ is bounded.

Techniques of integration: You should be able to evaluate integrals which are similar to the following examples:

(a) $\displaystyle\int_{-1}^{+4} \frac{x^2 + x + 1}{(2x^3 + 3x^2 + 6x + 5)^{2/3}}\, dx$

(b) $\displaystyle\int_0^{\pi} x^2 \cos(x^3)\, dx$

(c) $\displaystyle\int_0^{\pi} \sin(\theta) \cos^3(\theta)\, d\theta$

(d) $\displaystyle\int_0^{\pi} \sin^2(s)\, ds$

(e) $\displaystyle\int_0^{\frac{\pi}{2}} \cos^4(y)\, dy$

(f) $\displaystyle\int_{-\pi}^{+\pi} x \cos\left(\frac{x}{2}\right) dx$

(g) $\displaystyle\int_0^{\frac{\pi}{4}} \sin(u) \sec^5(u)\, du$

(h) $\displaystyle\int_1^{8} r^2 (1 - r^2)^3\, dr$

(i) $\displaystyle\int_0^{4} \sqrt{25 - t^2}\, dt$

(j) $\displaystyle\int_0^{10} \sqrt{1 + x^2}\, dx$

Differentiation: You should be able to use the first fundamental theorem of calculus to differentiate functions of the form $F(x) = \displaystyle\int_{a(x)}^{b(x)} f(t)\, dt$ to get

$$\frac{dF}{dx}(x) = f\big(b(x)\big)\, \frac{db}{dx}(x) - f\big(\big(a(x)\big)\, \frac{da}{dx}(x).$$