Math 275 **Review Sheet for Exam #2**

The second exam is on Thursday, November 9. The regular exam will be given from 5:30 to 7:00 PM in B139 Van Vleck. There will also be an early exam from 2:30 to 4:00 PM in B321 Van Vleck. You need to tell me or Jesse Holzer that you need to take the early exam. If you have a conflict with both exams, please arrange with one of us to take a make-up exam at a different time.

This handout is intended to help you to study for the exam. However, you may be tested on any of the material we have covered. Just because an item does not appear on this sheet, it does not mean that you will not be asked about it. In particular, you should know how to do the problems that have been assigned for homework.

Outline of topics:

Since the last exam we have studied the following topics. You should understand the concepts involved and how to apply them to various problems.

(1) The rules for differentiating sums, products, and quotients of functions.

(2) The chain rule.

(3) The characterization of differentiability in terms of the size of an error: Let f be defined on an interal (a, b) let $x \in (a, b)$, and let L be a real number. Write

$$f(x+h) = f(x) + h\,L + E_f(x,h).$$

Then f is differentiable at x and $f'(x) = L$ if and only if

$$\lim_{h \to 0} \frac{E_f(x,h)}{h} = 0.$$

(4) Related rate problems.

(5) Implicit differentiation.

(6) Rolle's theorem.

(7) The mean value theorem.

(8) The Cauchy mean value theorem.

(9) L'Hopital's rule.

(10) Estimating the error if we approximate $f(x+h)$ by $f(x)$ or by $f(x)+h\,f'(x)$.

(11) Tests for finding local extrema.

(12) The second derivative test.

(13) Problems that involve finding maxima and minima.

(14) Derivation of Snell's law.

(15) The axioms for area.

(16) The definition of the Riemann integral;

(17) Integrals of step functions;

(18) Elementary techniques for integration.

Definitions

You should know short but precise definitions of the following concepts:

(1) A local maximum, local minimum, and local extremum of a function f;
(2) A partition of an interval $[a, b]$;
(3) A step function;
(4) The integral of a step function;
(5) The upper and lower integral of a bounded function;
(6) What it means that a bounded function is Riemann integrable;

Proofs

You should know how to prove the following results:

(1) If f is continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b), and if f has a local extremum at a point $x \in (a, b)$, then $f'(x) = 0$.

(2) If f is continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b), and if $f(b) = f(a)$, then there exists at least one point $c \in (a, b)$ such that $f'(c) = 0$.

(3) Suppose $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$, and suppose there is a delta such that $g'(x) \neq 0$ for $0 < |x - a| < \delta$. Then if $\lim_{x \to a} \frac{f'(x)}{g'(x)} = L$, it follows that $\lim_{x \to a} \frac{f'(x)}{g'(x)} = L$.

(4) $\int_0^a x^n \, dx = \frac{1}{n+1} a^{n+1}$.

(5) If f is a bounded function on an interval $[a, b]$, and if f is monotone on that interval, then f is Riemann integrable on that interval.