Math 275 **Review Sheet for Exam #1**

The first exam is on Thursday, October 12. The regular exam will be given from 5:30 to 7:00 PM in B139 Van Vleck. There will also be an early exam from 2:30 to 4:00 PM in B321 Van Vleck. You need to tell me or Jesse Holzer that you need to take the early exam. If you have a conflict with both exams, please arrange with one of us to take a make-up exam at a different time.

The following outline should help you to study for the exam. However, you may be tested on any of the material we have covered. Just because an item does not appear on this sheet, it does not mean that you will not be asked about it. In particular, you should know how to do the problems that have been assigned for homework. However, we will not try to find obscure topics to trip you up. You should, as always, focus on the *BIG IDEAS*.

I Definitions You should know short but precise definitions of the following concepts:

(1) What it means that a set of real numbers has an *upper bound*.
(2) The statement of the *least upper bound axiom*.
(3) The *complex conjugate* $\bar{z}$ of a complex number z.
(4) What it means that *'the limit of* $f(x)$ *as* x *approaches* a *is* L*'* .
(5) You should be able to state the basic facts about limits of sums, differences, products, and quotients.
(6) What it means that a function f is *continuous* at a point a.
(7) You should be able to state *Bolzano's theorem* and the *Intermediate Value Theorem*.
(8) What it means that a function f is *differentiable* at a point a.
(9) What the *domain* of a function is.
(10) What the *range* of a function is.
(11) If f and g are two functions defined on the same set with real or complex values, how one defines the new functions $f+g$, $f-g$, fg, and f/g.
(12) If $f : X \to Y$ and $g : Y \to Z$ are two functions, how one defines the *composition* $g \circ f$.

You should also be able to tell whether some mathematical sentence has the same meaning as the definition of limit, as on problem #3 on the third homework set.

II Proofs You should be able to write down complete proofs of the following assertions:

(1) Numbers like $\sqrt{2}$ or $\sqrt[3]{5}$ are irrational.
(2) if $P(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$ is a polynomial with real coefficients, and if w is a complex number such that $P(w) = 0$, then $P(\bar{w}) = 0$ where $\bar{w}$ is the complex conjugate of z.
(3) If $\lim_{x \to a} f(x) = L$ and if $\lim_{x \to a} g(x) = M$, then $\lim_{x \to a} (f+g)(x) = L + M$.
(4) If $\lim_{x \to a} f(x) = L$ and if $\lim_{x \to a} g(x) = M$, then $\lim_{x \to a} (fg)(x) = LM$.

(5) If a function f is continuous at a and if $f(a) \neq 0$, then there exists an interval $(a-\delta, a+\delta)$ centered at a so that $|f(x)| > \frac{1}{2}|f(a)|$ for all $x \in (a-\delta, a+\delta)$.

(6) Prove the existence of limits directly from the definition. For example, you should be able to prove that $\lim_{x\to 4}(x^2 - 3x + 1) = 5$, $\lim_{x\to 3} \dfrac{2}{x+5} = \dfrac{1}{4}$, *etc.*

(7) Prove that if a function is differentiable at a point a, then it is continuous at a point a.

III Computations You should be able to do the following kinds of computations:

(1) Add, subtract, multiply, and divide complex numbers.

(2) Find the polar coordinates of a complex number.

(3) Find N^{th} roots of complex numbers.

(4) Find the limits of various explicit functions, and justify this by appealing to the basic limit properties.

(5) Find the derivatives of various functions such as polynomials and rational functions, and expressions involving $\sin(x)$ and $\cos(x)$.