Math 275 **Assignment # 7**

This is the last assignment of the semester. It is due at the final exam, which will be given on Tuesday, December 19, from 12:25 – 2:25 PM in B239 Van Vleck Hall.

In the text, read Chapter 6, sections 1 – 8, 10, 12 –16, and 20 – 21, and do the following problems:

Problem 1: In section 6.9 on pages 236 – 238, do problems 2, 3, 5, 7, 8, 12, 16, 17, 19, 22, 28, and 31.

Problem 2: In section 6.17 on pages 248 – 250, do problems 3, 4, 5, 6, 9, 10, 14, 15, 17, 25, 29, 30, 39, and 40.

Problem 3: In section 6.22 on pages 256 – 258, do problems 13, 18, 21, 25, 30, 33, 36, 40, 43, 45.

Problem 3: Define the function $AT(x)$ by the formula

$$AT(x) = \int_0^x \frac{1}{1+t^2}\,dt$$

and define the real number π by the equation

$$\frac{\pi}{2} = \lim_{x\to+\infty} AT(x) = \int_0^{+\infty} \frac{1}{1+t^2}\,dt.$$

(a) Show that $AT\left(\frac{1}{x}\right) = \frac{\pi}{2} - AT(x)$.

(b) What is the geometric interpretation of the identity in part (a)?

(c) Show that if $uv \neq 1$

$$AT\left(\frac{u+v}{1-uv}\right) = AT(u) + AT(v).$$

Hint: In the integral definining $AT\left(\frac{u+v}{1-uv}\right) = \int_0^{\frac{u+v}{1-uv}} \frac{1}{1+t^2}\,dt$, make the change of variables $t = \frac{s+v}{1-sv}$.

(d) If we define the function $y = \tan(\theta)$ for $\theta \in \left(-\frac{\pi}{2}, +\frac{\pi}{2}\right)$ as the inverse function to the function AT, show that

$$\tan(\theta_1 + \theta_1) = \frac{\tan(\theta_1) + \tan(\theta_2)}{1 - \tan(\theta_1)\tan(\theta_2)}.$$