Math 275 **Assignment # 3**

Due Thursday, October 12, 2006

In the text, read Chapter 3, sections 7, 9, and 10, and Chapter 4, sections 1 – 5. Do the following problems:

Problem 1 Use the formal mathematical definition of limit to show each of the following. **Do not just use the limit theorems to plug in!**

(a) $\lim_{x \to -2} (3x + 7) = 1;$

(b) $\lim_{t \to 3} (t^2 - 2t) = 3;$

(c) $\lim_{y \to 4} \dfrac{1}{y - 3} = 1.$

Problem 2 If a is a real or complex number and f is a function, we have given a precise definition for the idea that the limit of $f(x)$ as x approaches a is L. Sometimes, we want to talk about limits as the variable x goes off to positive or negative infinity. If f is a function defined for all real numbers, write down what you think is a good formal mathematical definition of the expressions

$$\lim_{x \to +\infty} f(x) = L \qquad \text{and} \qquad \lim_{x \to -\infty} f(x) = M.$$

Problem 3 Recall that the definition of $\lim_{x \to a} f(x) = L$ is:

$$\boxed{\begin{array}{l}\textit{For every } \epsilon > 0, \textit{ there exists a } \delta > 0 \textit{ so that if} \\ 0 < |x - a| < \delta, \textit{ it follows that } |f(x) - L| < \epsilon.\end{array}}$$

In each of the following, there is a mathematical sentence which sounds a bit like this definition. In each case, decide if the sentence means the same thing as the definition of limit. If not, explain why the sentence means something different:

(a) For every $\epsilon > 0$ and for every $\delta > 0$, if $0 < |x - a| < \delta$, it follows that $|f(x) - L| < \epsilon$.

(b) There exists a positive number $\delta > 0$ so that for every $\epsilon > 0$, if $0 < |x - a| < \delta$, it follows that $|f(x) - L| < \epsilon$.

(c) For every $\epsilon > 0$, there exists a $\delta > 0$ so that if $|f(x) - L| \geq \epsilon$, it follows that either $x = a$ or $|x - a| \geq \delta$.

(d) For every $\epsilon > 0$, there exists a $\delta > 0$ so that if $|f(x) - L| < \epsilon$, it follows that $0 < |x - a| < \delta$.

Problem 4 Suppose that on Monday I drive along a one-lane road from point A to point B, starting at 9:00 AM, and ending at 1:00 PM. On Tuesday, I take the same road, and drive back from point B to point A, again starting at 9:00 AM and ending at 1:00 PM. Must it be true that there is a time on *both* days when I am at exactly the same place on the road? I do not necessarily drive at a constant speed, or even always go in the same direction. (Hint: Can you apply the intermediate value theorem?)

Problem 5 Let $P(x) = a_0 + a_1 x + \cdots + a_n x^n$ be a polynomial of degree n with real coefficients. Suppose that the coefficients a_0 and a_n are both non-zero, and that they have opposite signs. Prove that the equation $P(x) = 0$ has at least one positive root; *i.e.*, there is at least one real number $c > 0$ such that $P(c) = 0$.

Problem 6 Do problem 2 on page 167 in the text.

Problem 7 In each of the following, find the derivative $f'(x)$ of the given function $f(x)$.

(a) $f(x) = \dfrac{1}{x^2+4} + 9x^3.$

(b) $f(x) = x^2 \cos(x);$

(c) $f(x) = \dfrac{x^3+x^2+x}{x^4+3x^2+1};$

(d) $f(x) = \dfrac{x\sin(x)+\cos(x)}{x\cos(x)-\sin(x)}.$

Problem 8 Do problem 13 on page 167 in the text.

Problem 9 Do problem 15 on page 167 in the text.

Problem 10 Do problem 38 on page 168 in the text.