Math 275 **Assignment # 2**

Due Tuesday, September 26, 2006

Read Chapter 1, sections 1 – 4, and Chapter 3, sections 1 – 5 in the text, and do the following problems:

Problem 1 Let $f(x) = \dfrac{x+1}{x-1}$, and let $g(x) = +\sqrt{x-1}$.

(a) What are the domains and ranges of the functions f and g?

(b) Compute each of the following: $(f+g)(2)$, $(fg)(2)$, $g\big[f(2)\big]$.

(c) What is the function $h = (f-1)g^2$?

Problem 2 Let $P(z) = \sum_{k=0}^{n} c_k z^k$ be a polynomial of degree n, where the coefficients are complex numbers and $c_n \neq 0$. Prove each of the following statements:

(a) If $n \geq 1$ and $P(0) = 0$ then $c_0 = 0$ and $P(z) = zQ(z)$ where Q is a polynomial of degree at most $n-1$.

(b) For each complex number $a \in \mathbb{C}$, the function given by $f(z) = P(z+a)$ is also a polynomial of degree n.

(c) If $n \geq 1$ and $P(a) = 0$ for some complex number $a \in \mathbb{C}$, then $P(z) = (z-a)R(z)$ where R is a polynomial of degree at most $n-1$. [Hint: apply part (a) to the polynomial f of part (b)].

(d) If $P(z) = 0$ for $n+1$ distinct complex values of z, then every coefficient $c_k = 0$, and hence $P(z) = 0$ for *all* complex numbers z.

Problem 3 For each of the following, find *all* complex polynomials $P(z)$ of degree ≤ 2 which satisfy the given conditions:

(a) $P(0) = P(i) = P(3-17i) = 5$.

(b) $P(1) = P(-i) = 1$.

(c) $P(1) = P(i)$ and $P(1+i) = 2$.

(d) $P(z) = P(1-z)$.

(e) $P(iz) = iP(z)$.

Problem 4 In each of the following, compute the limit, and explain which theorems about limits you are using:

(a) $\displaystyle\lim_{x\to 3} \frac{1}{(x-2)^2}$; (b) $\displaystyle\lim_{x\to 2} \frac{x^2-4}{x-2}$.

(c) $\displaystyle\lim_{h\to 0} \frac{(t+h)^3 - t^3}{h}$ (d) $\displaystyle\lim_{x\to a} \frac{x^2 - 2ax + a^2}{x^2 - a^2}$.

(e) $\displaystyle\lim_{t\to 0^+} \frac{|t|}{t}$. (f) $\displaystyle\lim_{t\to 0^-} \frac{|t|}{t}$.

Problem 5 Define a function $f : \mathbb{R} \to \mathbb{R}$ as follows:

$$f(x) = \begin{cases} \sin(3x) & \text{if } x \leq c, \\ ax + b & \text{if } x > c, \end{cases}$$

where a, b, c are real constants. If b and c are given, find all values of a for which f is continuous at the point $x = c$.

Problem 6 For all real numbers $x \neq 0$, define $g(x) = \sin\left(\frac{1}{x}\right)$. Show that there is **no** real number A such that $\lim_{x\to 0} g(x) = A$. [Hint: Look at the discussion in problem 27 on page 139 of the text].

Problem 7 For all real numbers $x \neq 0$, define $h(x) = x \sin\left(\frac{1}{x}\right)$. Can h be defined at $x = 0$ so that the function is continuous at $x = 0$?

Problem 8 Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ which is continuous at $x = 0$ but is not continuous at any other point $a \in \mathbb{R}$.

Problem 9 Let f be a real-valued function defined on an interval $[a, b] \subset \mathbb{R}$. Suppose that $|f(\alpha) - f(\beta)| \leq |\alpha - \beta|$ for all $\alpha, \beta \in [a, b]$. Prove that f is continuous at every point $x \in [a, b]$.

Problem 10 Write out in detain the proof that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a} (fg)(x) = LM$.