Math 751 – Kent – Midterm

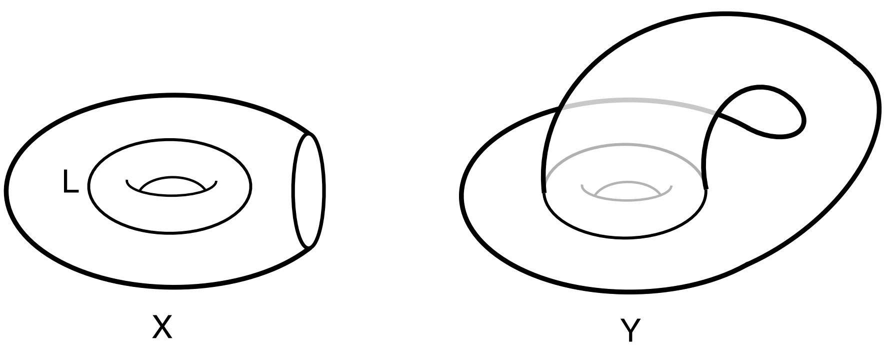
due 10/26/12 by noon.

YOUR NAME:____________________________________

Rules: You may not use the Internet. You may not discuss the problems with each other.

1. Let X be a compact surface with nonempty boundary. Show that $\pi_1(X)$ is a free group. If Y is a noncompact surface without boundary, show that $\pi_1(Y)$ is free.

2. Let X be the compact surface with boundary shown in the figure. It is obtained by deleting an open ball from the torus, and it has a single boundary component. Let L be the loop pictured. Let Y be the space obtained by identifying ∂X and L as pictured. Compute $\pi_1(Y)$. (Give a presentation.) What is the abelianization of $\pi_1(Y)$?



3. Let X and Y be as in the previous problem. The space Y contains a subspace Z homeomorphic to X, obtained by homotoping the quotient map $X \to Y$ to an embedding. (Alternatively, you may imagine Z as the image under the quotient map $X \to Y$ of X minus a neighborhood of ∂X.) Is Z a retract of Y? Prove your assertion.

4. (Chapter 0, number 25.) Let X be a cell complex whose components are X_α. Show that the suspension SX is homotopy equivalent to $Y \vee (\bigvee_\alpha SX_\alpha)$ for some graph Y. If X is a finite graph, show that SX is homotopy equivalent to a wedge sum of circles and 2–spheres.

5. (1.2, number 4.) Let X be the union of n lines through the origin in $\mathbb{R}^3$. Compute $\pi_1(\mathbb{R}^3 - X)$.