Do $\boxed{4}$ problems.

Do [illegible]

A. Set Theory

1. Let κ be the least regular cardinal such that for all $\alpha < \kappa$, $\kappa \to (\alpha)^{<\omega}$. Prove that κ is not Mahlo.

2. Let κ be the least inaccessible cardinal such that $\{\alpha < \kappa : \alpha \text{ inaccessible}\}$ is stationary in κ. Show that κ is not measurable.

3. Suppose M is a countable transitive model of ZFC. Show that there are transitive models N_1, N_2 for ZFC such that $M \neq N_1$, $M \neq N_2$, and $M = N_1 \cap N_2$

B. Model Theory

4. Prove that for every regular cardinal κ and every model $\langle M, E\rangle$ of ZF, $\langle M, E\rangle$ has an elementary extension $\langle M_1, E_1\rangle$ such that ~~the~~ (ordinals of $M_1, E_1\rangle$) has cofinality κ.

5. Let $T = \bigcup_n T_n$ be a first order theory in a countable language, with $T_0 \subseteq T_1 \subseteq T_2 \subseteq \cdots$. Let U be a unary relation of T. Suppose that for each n there is a κ such that T_n has a model $\mathfrak{A}_n = \langle A_n, U_n, \ldots\rangle$ with $|A_n| = \beth_n(\kappa)$ and $|U_n| = \kappa$. Prove that T has a model $\mathfrak{A} = \langle A, U, \ldots\rangle$ with $|A| = \beth_\omega$, $|U| = \omega$.

C. Recursion Theory

6. Show that $\{e : e$ is the Gödel number of a total function$\}$ is not Σ^0_2

7. $A \subset \omega$ is a spectrum iff there is a first order sentence φ such that $A = \{n \in \omega : \varphi$ has a model of power $n\}$. Show that there is a recursive A which is not a spectrum.