A. Model Theory

A1. Let $T$ be the theory of all models $(A,E)$ where $E$ is an equivalence relation. Prove that $T$ is $\omega$-stable.

A2. Let $T$ be a theory in a countable language. Suppose that for some infinite cardinal $\kappa$, every model of $T$ of power $\kappa$ is atomic. Prove that every model of $T$ is atomic.

A3. Prove that every infinite saturated model has a proper elementary submodel to which it is isomorphic.

**: Give an example of a model $\mathcal{M}$ for a countable language such that $\mathcal{M}$ has power $\omega_1$ but every proper elementary submodel of $\mathcal{M}$ is countable.

B. Set Theory.

B1. Let $N$ be a transitive class containing all the ordinals, such that for each $\alpha$, $N \cap \mathbb{R}(\alpha) = N$. Assume that $(N,\in)$ satisfies the comprehension axiom scheme. Prove that $(N,\in)$ is a model of ZF.

B2. Assume the axiom of choice and that the union of fewer than $2^\omega$ sets of reals of Lebesgue measure 0 is of Lebesgue measure 0. Prove that $2^\omega$ is regular.

B3. Outline a proof of the consistency of Luzin's hypothesis ($2^\omega = 2^{\omega_1}$) with the axioms of ZFC.

B4. Assume that ZF is consistent. Show that there is a finite subtheory $T$ of ZF such that in ZF it cannot be proved that $T \cup \text{"there is an uncountable inaccessible cardinal."}$ is consistent.
C. Recursion Theory.

C1. Let $T$ be a recursively axiomatized theory in a countable language such that $T$ is $\mathbb{K}_0$-categorical. Prove that $T$ has a recursive model.

C2. Let $A$ be a $\Pi^1_1$ subset of $\omega$. Show that either $\theta'$ is hyperarithmetical in $A$ or $A$ is hyperarithmetical.

C3. Show that there is a sequence $f_\alpha : \alpha < \omega_1$ of functions mapping $\omega$ into $\omega$ such that whenever $\alpha < \beta < \omega_1$, $f_\alpha$ is recursive in $f_\beta$ but $f_\beta$ is not recursive in $f_\alpha$.

C4. Let $\{\varphi_0, \varphi_1, \varphi_2, \ldots\}$ be an r.e. set of sentences of first order logic (i.e., the set of Gödel numbers is r.e.). Prove that there is a recursive set of sentences $\{\psi_0, \psi_1, \psi_2, \ldots\}$ such that for each $n$, $\psi_n$ is logically equivalent to $\varphi_n$.

\[ \psi_n = \varphi_n \]