Don't Forget the Hard Old Questions!

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April 27, 2021

Disclaimer: None of these questions are mine!

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Some have remained open for a very long time but are worth keeping in mind.
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- Lachlan? (late 1960's): Which finite lattices can be embedded into the c.e. Turing degrees? (Lerman 2000 gives Π^0_2-criterion for join-semidistributive lattices)

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- Late 1960's (outgrowth of homogeneity problem): Which natural degree structures have nontrivial automorphisms? (Cf. Ershov/Palyutin 1975 for *m*-degrees, Denisov 1978 for c.e. *m*-degrees, Slaman/Woodin 1990's for Turing degrees and hyperarithmetic degrees, Slaman/M. Soskova 2017-18 for enumeration degrees)

- Ershov (1977): For which finite families F_1 and F_2 of c.e. sets are the Rogers semilattices $R(F_1)$ and $R(F_2)$ isomorphic? The Rogers semilattice of F is the upper semilattice of all uniformly computable enumerations $\{V_e\}_{e \in \omega}$ modulo computable equivalence.
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- An. Muchnik, Semënov, Uspensky (1998): Do Martin-Löf randomness and Kolmogorov-Loveland randomness coincide? (Kolmogorov-Loveland randomness is defined in terms of computable, non-monotonic, adaptive martingales.
  Cf. Kastermans/Lempp 2010 separating Martin-Löf and injective randomness)

Ideas for answering them are welcome!