

MATRICES and LINEAR ALGEBRA

second edition

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preface

to the second edition

The primary difference between this new edition and the first one is the addition of several exercises in each chapter and a brand new section in Chapter 7. The exercises, which are both true-false and multiple choice, will enable the student to test his grasp of the definitions and theorems in the chapter. The new section in Chapter 7 illustrates the geometric content of Sylvester's Theorem by means of conic sections and quadric surfaces.

We would also like to thank the correspondents and students who have brought to our attention various misprints in the first edition that we have corrected in this edition.

MADISON, WISCONSIN
KANSAS CITY, MISSOURI
OCTOBER 1972

*H. S.
G. P. B.*



preface

to the first edition

Linear algebra is now one of the central disciplines in mathematics. A student of pure mathematics must know linear algebra if he is to continue with modern algebra or functional analysis. Much of the mathematics now taught to engineers and physicists requires it. It is for this reason that the Committee on Undergraduate Programs in Mathematics recommends that linear algebra be taught early in the undergraduate curriculum. In this book, written mainly for students in physics, engineering, economics, and other fields outside mathematics, we attempt to make the subject accessible to a sophomore or even a freshman student with little mathematical experience. After a short introduction to matrices in Chapter 1, we deal with the solving of linear equations in Chapter 2. We then use the insight gained there to motivate the study of abstract vector spaces in Chapter 3. Chapter 4 deals with determinants. Here we give an axiomatic definition, but quickly develop the determinant as a signed sum of products.

For the last thirty years there has been a vigorous and sometimes acrimonious discussion between the proponents of matrices and those of linear transformation. The controversy now appears somewhat absurd, since the level of abstraction that is appropriate is surely determined by the mathematical goal. Thus, if one is aiming to generalize toward ring theory, one should evidently stress linear transformations. On the other hand, if one is looking for the linear algebra analogue of the classical inequalities, then clearly matrices

form the natural setting. From a pedagogical point of view, it seems appropriate to us, in the case of sophomore students, first to deal with matrices. We turn to linear transformations in Chapter 5. In Chapter 6, which deals with eigenvalues and similarity, we do some rapid switching between the matrix and the linear transformation points of view. We use whichever approach seems better at any given time. We feel that a student of linear algebra must acquire the skill of switching from one point of view to another to become proficient in this field.

Chapter 7 deals with inner product spaces. In Chapter 8 we deal with systems of linear differential equations. Obviously, for this chapter (and this chapter only) calculus is a prerequisite. There are at least two good reasons for including some linear differential equations in this linear algebra book. First, a student whose only model for a linear transformation is a matrix does not see why the abstract approach is desirable at all. If he is shown that certain differential operators are linear transformations also, then the point of abstraction becomes much more meaningful. Second, the kind of student we have in mind must become familiar with linear differential equations at some stage in his career, and quite often he is aware of this. We have found in teaching this course at the University of Wisconsin that the promise that the subject we are teaching can be applied to differential equations will motivate some students strongly.

We gratefully acknowledge support from the National Science Foundation under the auspices of the Committee on Undergraduate Programs in Mathematics for producing some preliminary notes in linear algebra. These notes were produced by Ken Casey and Ken Kapp, to whom thanks are also due. Some problems were supplied by Leroy Dickey and Peter Smith. Steve Bauman has taught from a preliminary version of this book, and we thank him for suggesting some improvements. We should also like to thank our publishers, Holt, Rinehart and Winston, and their mathematics editor, Robert M. Thrall. His remarks and criticisms have helped us to improve this book.

MADISON, WISCONSIN
JANUARY 1968

H.S.
G.P.B.

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