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Corrections and Additions to: "Principal Components of Minus M-Matrices"

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Professor Bit-Shun Tam has pointed to us that the statement of Lemma 2 and the proof of Lemma 1 in our paper which appeared in Vol. 32: 131–148, 1992, need correction and augmentation, respectively. We begin with the correction:

LEMMA 2 *Let A be a minus M-matrix as in* (2.1) *and let i and j be vertices in $\mathcal{R}(A)$. Then* $Z^{(d(i,j)-1)}[\{i,j\}] > 0$.

Proof First, by the Rothblum index theorem we have that $d(i,j) = \nu(A\{i,j\})$. Thus the result is a consequence of Lemma 1(i) and of the resolvent expansion of $A\{i,j\}$ which, in a sufficiently small punctured neighborhood of 0, satisfies that $(\epsilon I - A\{i,j\})^{-1} \geq 0$, $\forall\ \epsilon > 0$. ■

Next, we wish to clarify the proof of the latter part of Lemma 1(i) in which we claim that $Z^{(k)}[\langle i,j\rangle] = (A\{i,j\})^k Z_{A\{i,j\}}$. First it is a simple consequence of the first part of the claim that if q is any polynomial such that $q(A) = 0$, then $q(A\{i,j\}) = 0$. Whence, for every complex z such that $(zI - A)^{-1}$ exists, $(zI - A)^{-1}[\{i,j\}] = (zI - A\{i,j\})^{-1}$. We now express the resolvents of A and of $A\{i,j\}$ in terms of the principal components corresponding to their eigenvalues λ and we compare coefficients of $(z-\lambda)^{-s}$. It follows that $Z^{(k)}[\langle i,j\rangle] = (A\{i,j\})^k Z_{A\{i,j\}}$.

The proof we give in the paper for Corollary 1 establishes the weaker result below (and we do not know if Corollary 1 as stated originally is correct).

COROLLARY 1 *Suppose A is a minus M-matrix given in form* (2.1). *If, for sufficiently small $\epsilon > 0$, a basis can be extracted for the columns of J given in* (3.7) *which satisfies* (3.9), *where $c_{k,j} \geq 0$, $k, j = 1, \ldots, m$, then* (3.10) *holds.*

In fact, Corollary 1 as stated here can also be deduced from the more general result proved in [6, Cor. (3.17)].

We have not found a counter-example to Corollary 1 as stated originally. However, in what follows we give here an example which shows that an arbitrary choice of columns of J may yield a strongly combinatorial basis for the Perron space of A which however is not a semi-preferred basis:

Let

$$A = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}.$$

Then, with $\epsilon = 1/2$,

$$8J = \begin{pmatrix} 8 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 8 & 0 & 0 & 0 & 0 \\ 28 & 36 & 0 & 0 & 16 & 0 & 0 & 0 \\ 36 & 28 & 28 & 36 & 0 & 16 & 0 & 0 \\ 142 & 142 & 63 & 79 & 36 & 36 & 8 & 8 \\ 114 & 114 & 51 & 63 & 28 & 28 & 8 & 8 \end{pmatrix}.$$

Let B be the matrix obtained by choosing columns 1, 3, 5, 6, and 7 of J. Then the columns of B form a strongly combinatorial basis (in the sense of the paper) for the Perron space of A. However the matrix C which satisfies $AB = BC$ (and which therefore contains the coefficients $c_{k,j}$ of (3.9)) is given by:

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & -1/4 & 1 & 1 & 0 \end{pmatrix}.$$

Since C has a negative entry, the basis given by B is not semi-preferred. Whereas, on choosing columns 1, 4, 5, 6, and 7 the matrix C so obtained is

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 1/4 & 1 & 1 & 0 \end{pmatrix}.$$

Thus the columns of B are a semi-preferred basis for the Perron space of A.

We have a further relevant comment. The matrix J depends on a choice of ϵ. However, for any fixed choice of columns of J which form a basis B for the Perron space of A, it can be shown that the induced matrix C satisfying $AB = BC$ is independent of ϵ.

We are very grateful to Professor Tam for spotting the necessity for the above corrections.