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The diagonal equivalence of a nonnegative matrix to a stochastic matrix.

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An n-square matrix with non-negative entries is fully indecomposable if there do not exist permutation matrices P and Q such that $PAQ = \begin{bmatrix} A_1 & 0 \\ B & A_2 \end{bmatrix}$ with A_1 and A_2 square matrices. This paper contains a proof of the following theorem which was obtained independently by R. Sinkhorn and P. Knopp [Pacific J. Math. **21** (1967), 343–348]. Theorem: If A is an n-square non-negative fully indecomposable matrix, then there exist diagonal matrices D_1 and D_2 with positive main diagonal entries such that D_1AD_2 is doubly stochastic. Moreover, D_1 and D_2 are unique to within scalar multiples. An interesting corollary of a related result states that if A has positive main diagonal entries then there exists a diagonal matrix D with positive main diagonal entries such that DAD is row stochastic. The approach to this problem is to analyze an operator associated with A introduced by M. V. Menon [Proc. Amer. Math. Soc. **18** (1967), 244–247]. This is defined as follows: If x is a non-negative n-tuple, define Tx to be the non-negative n-tuple whose ith component is $(\sum_{j=1}^n a_{ji}(\sum_{k=1}^n a_{jk}x_k)^{-1})^{-1}$. (The usual conventions $0^{-1} = \infty$, $\infty^{-1} = 0$, etc., are made.) The properties of T regarded as a non-linear operator on the set of non-negative vectors are developed. In particular, the existence of fixed points of T with positive components is clearly related to the problem of reducing A to a doubly stochastic matrix by diagonal equivalence.

Reviewed by *M. Marcus*